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UV finite brane decay

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Abstract: The decay of an unstable D-brane via closed string emission and open string pair production is considered in subcritical string theory with a spacelike linear dilaton. The decay rate is given by the imaginary part of the annulus, which has ambiguities corresponding to the choices of incoming closed and open string vacua. An exact expression for the full annulus diagram is computed with a natural choice of incoming vacua. It is found that the ultraviolet divergences present in critical string theory in both of these processes are absent for any nonzero spacelike dilaton. Implications for the vexing issue of the tachyon dust are discussed.

Keywords: Brane Dynamics in Gauge Theories, Tachyon Condensation.

Contents

1. Introduction

The study of s-branes, or time-dependent open string tachyons, has led to several interesting puzzles. These puzzles are technically challenging and their resolutions may lead to new conceptual insights. Recent work includes [1]–[14].

The first puzzle is the appearance of the so-called tachyon dust [2]. Open string tachyon condensation, or equivalently brane decay, can be described by an exact worldsheet CFT. Using this description Sen [2] argued that, for $g_s = 0$, at the endpoint of tachyon condensation the unstable brane is replaced by a pressureless gas of "tachyon dust". This seems peculiar because, once the tachyon fully condenses, the brane should disappear and one expects no open string excitations. Rather, all excitations should be described in terms of closed strings. This raises the question: Is the tachyon dust closed strings in disguise, or a new form of matter in string theory?

It was subsequently suggested [4] that the appearance of the tachyon dust is an unphysical artifact of perturbation theory which does not survive to any finite, nonzero g_s. The time-dependent tachyon background leads to open string pair production with Hagedorn-like divergences. Hence at any finite value of g_s perturbation theory will break down in a time of order one in string units, long before the tachyon dust is encountered, and the brane energy is presumably converted into outgoing closed strings. In [13] a similar Hagedorn-like divergence was found in closed string emission, and was also argued to lead to the breakdown of perturbation theory and the conversion of all brane energy into outgoing closed strings. It was further suggested [13, 15, 16] that the tachyon dust might actually be comprised of the highly massive closed strings emitted during the decay process.

While it may seem comforting that the breakdown of perturbation theory saves us from a direct confrontation with the tachyon dust, in this paper we will find that both the open and closed string divergences are controlled in a subcritical string theory with a spacelike linear dilaton. The result is finite[1] (per unit spatial volume) expressions for both the closed string emission and the open string pair production. The basic reason for this is that subcritical string theory has a higher Hagedorn temperature. In this setting perturbation theory does not seem to break down, and the energy in closed string emission is insufficient (by a factor of g_s) to account for the energy in the tachyon dust as proposed in [13, 15, 16].[2] The dust typically drifts into the weak coupling region where it might be perturbatively studied. Hence the tachyon dust puzzle has been at least temporarily resuscitated, although we are confident that this paper will not be the last word on the subject.

The main result of this paper, on which the preceding statements are based, is the expression for the worldsheet partition function for unstable for p-brane decay in $D \leq 26$-dimensional string theory

$$Z\left(\frac{is}{2\pi}\right) = \frac{2V_p}{(4\pi s)^{\frac{p+2}{2}}} \int_{-\infty}^{\infty} d\nu e^{-s\nu^2} \eta^{2-D}\left(\frac{is}{2\pi}\right) \partial_\nu \ln S_\beta\left(\frac{\beta}{2} + \frac{1}{2\beta} - \nu\right), \tag{1.1}$$

where the dilaton gradient has a timelike component $\partial_0 \Phi = \beta - \frac{1}{\beta}$, S_β is the q-gamma function, s is the modular parameter of the annulus and the ν integration contour is just above (below) the positive (negative) real axis. Z acquires an imaginary part from integrating past the poles given by

$$\operatorname{Im} Z\left(\frac{is}{2\pi}\right) = V_p \sum_{m,n=0}^{\infty} \frac{\eta^{2-D}\left(\frac{is}{2\pi}\right)}{s(4\pi s)^{\frac{p}{2}}}\, e^{-\left((n+\frac{1}{2})\beta + (m+\frac{1}{2})/\beta\right)^2 s}. \tag{1.2}$$

Interpreting the imaginary part as due to closed string emission, this agrees with the result of [13] for closed string emission during brane decay, and generalizes it to the case of a linear dilaton. When $D \to 26$, $\beta \to 1$ and the emission is divergent. We show that for $D < 26$ this divergence is eliminated. Though this divergence is gone in all the different decay channels which we have considered, it is possible that there exist other decay channels where the emission rate diverges again.

This result sharpens the tachyon dust puzzle while underscoring a second puzzle. We are used to the idea that classical solutions of string theory correspond to worldsheet conformal field theories, which in turn have well defined partition functions and annulus diagrams. However it is clear that life cannot be quite so simple in the time dependent context. Formally the annulus diagram describes both open string pair production and closed string emission. However these are *not* determined by the classical solution alone: the incoming state of both the open and closed strings must be specified. Hence there cannot be a uniquely defined annulus diagram for every classical solution of string theory. In practice

[1]Up to certain tachyon divergences which are presumably absent in the full s-brane of the superstring, as discussed in section 4.2.

[2]However in section 3 we will find some evidence supporting the idea that the tachyon dust is composed of massive closed strings (of unknown origin).

JHEP11(2003)042

what happens is that the CFTs which describe time dependent string backgrounds are a priori ill-defined because the worldsheet fields have negative kinetic terms. A prescription must be introduced — often some kind of analytic continuation or contour — for defining them. Hence in practice there may be more than one set of correlators/CFT corresponding to a given classical solution.[3] It has been suggested [18] that this mathematical ambiguity in the CFT corresponds to the physical ambiguity in the choice of vacua.

At present we have no detailed understanding of how these ambiguities match up. The case at hand provides a concrete example in which this issue can be addressed. The annulus diagram has an imaginary part which can be identified, in the closed string channel, as coming from on-shell closed string emission. On the other hand, the contribution in the open string channel from on-shell open strings is purely real, and there is no obvious signal of decay via open string pair production. One possible explanation is that the prescription adopted herein for the annulus corresponds to the usual Minkowski vacuum state for the incoming closed strings, and to the *out* vacuum for the incoming open strings, for which there are no outgoing open strings and hence no associated imaginary part.[4] A second possibility is that the annulus needs a second term containing the representations corresponding to on-shell open strings. Yet a third possibility is that the closed string emission has a dual channel interpretation as open string pair production. Our calculations give some insight into this problem. These possibilities do not all seem equally likely but we will not arrive at a definitive conclusion.

Some interesting insights into these issues have recently been obtained in the context of $c = 1$ string theory [19, 20, 21].

This paper is organized as follows. In section 2, we review some facts about the physics in the linear dilaton background. In section 3 we compute the stress-energy tensor for a brane decaying in this background, and derive a modified conservation law. In section 4, we define and compute both the real and imaginary parts of the annulus in this background and show that it is finite (up to the usual tachyon divergences). We also show in the open string channel that (1.1) is real (even after integrating over the modular parameter) near the massive on-shell open states where open string pair production could show up. In section 5 we compute the asymptotic form of open string pair production in the *in* vacuum, and again find a UV finite result when there is a spacelike linear dilaton.

Throughout this paper, we take $\alpha' = 1$.

2. Strings in the linear dilaton background

In this section, we discuss the physics of free strings in the linear dilaton background [22].

Consider string theory in flat space with a linear dilaton $\Phi = V_\mu X^\mu$. The linear dilaton worldsheet CFT differs from the regular free boson CFT only in the quantum improvement

[3] An example of this was recently discussed in [17].

[4] The Minkowski closed string vacuum arises naturally in the contour prescription of [13] for half s-branes, while the *out* vacuum for open strings arises naturally in the analytic continuation from the positive-norm Liouville theory [18].

term for the energy momentum tensor [23]

$$T(z) = -\partial X^\mu \partial X_\mu + V_\mu \partial^2 X^\mu\,, \tag{2.1}$$
$$T(\bar{z}) = -\bar{\partial} X^\mu \bar{\partial} X_\mu + V_\mu \bar{\partial}^2 X^\mu\,. \tag{2.2}$$

The central charge is modified to be $c = \tilde{c} = D + 6V^2$, where D is the number of spacetime directions and $V^2 \equiv V_\mu V^\mu$. For nonzero V, D must be adjusted to maintain $c = 26$.[5] The mass-shell conditions for closed strings are

$$-(p + iV)^2 = 2(N + \tilde{N} - 2) + V^2\,, \tag{2.3}$$

together with the level matching condition $N = \tilde{N}$. On-shell momenta thus have an imaginary component, $-iV_\mu$. This is natural when one examines, for example, the lagrangian for a closed string mode of mass $m^2 = 2(N + \tilde{N} - 2)$ in this background

$$\mathcal{L}[\varphi] = e^{-2V_\mu X^\mu}\left(-\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m^2\varphi^2\right). \tag{2.4}$$

Defining a new field, $\tilde{\varphi} \equiv e^{-V_\mu X^\mu}\varphi$, we obtain

$$\mathcal{L}[\tilde{\phi}] = -\eta^{\mu\nu}\partial_\mu\tilde{\varphi}\partial_\nu\tilde{\varphi} - (m^2 + V^2)\tilde{\varphi}^2 + \text{total derivative}\,, \tag{2.5}$$

giving $\tilde{\varphi}$ a mass squared

$$\tilde{m}^2 = 2(N + \tilde{N} - 2) + V^2\,. \tag{2.6}$$

We can refer to φ as the string frame field. The string frame stress-energy tensor is

$$T_{\mu\nu} \equiv \frac{\delta S}{\delta g^{\mu\nu}} = e^{-2\Phi}\left[\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\eta_{\mu\nu}((\partial\varphi)^2 + m^2\varphi^2)\right], \tag{2.7}$$

and it obeys a modified conservation law

$$\partial^\mu T_{\mu\nu} = -V_\nu \mathcal{L} = \frac{1}{2}V_\nu \frac{\delta S}{\delta\Phi}\,. \tag{2.8}$$

We will refer to the vertex operators of the from $e^{ik_\mu X^\mu + V_\mu X^\mu}$ as the dressed vertex operators. As seen above, the dressing is necessary to make the physical field normalizable with respect to the norm given by the action.

For the open string spectrum the arguments are similar. However, since the open string action has only one power of string coupling in front of it, the open string dressed operators are of the form $e^{ik_\mu X^\mu + \frac{1}{2}V_\mu X^\mu}$. The mass-shell condition is

$$-\left(p + \frac{i}{2}V\right)^2 = (N-1) + \frac{1}{4}V^2\,. \tag{2.9}$$

Since the number of string oscillators is proportional to $(D-2)$, the Hagedorn temperature becomes

$$T_H^{(D)} = \frac{1}{2\pi\sqrt{\frac{D-2}{6}}} = \frac{1}{2\pi\sqrt{(4 - V^2)}}\,. \tag{2.10}$$

For subcritical string theory (spacelike V_μ), this is higher than for $V_\mu = 0$.

[5] We assume $6V^2$ is an integer so that this is possible, although the more general case is easily treated using more general CFTs.

3. Stress-energy of subcritical brane decay

In this section we begin our analysis of a D-brane decaying in the linear-dilaton background by computing the stress-energy, generalizing the computation of [2].

Tachyon condensates on an unstable brane can be of the general form $e^{t_\mu X^\mu}$. However for the most part we shall find it convenient to boost into a frame which brings this into the form $\exp(\beta X^0)$, with $\beta > 0$. We want to study this in an arbitrary linear dilaton of the form $\Phi = V_\mu X^\mu$. We require the dilaton gradient to point along the unstable brane (in order to avoid a force on the brane), which, if the dilaton is spacelike, is possible only for $p > 0$. The weight of the worldsheet boundary interaction operator is $\beta(\beta - V_0)$. Requiring this to be 1, so the interaction is marginal, we obtain

$$V_0 = \beta - \frac{1}{\beta}\,. \tag{3.1}$$

The timelike part of the worldsheet CFT is the TBL (Timelike Boundary Liouville) theory

$$-\frac{1}{2\pi}\int_\Sigma \left(\partial X^0 \bar\partial X^0 - \frac{V_0 X^0 R}{4}\right) + \frac{1}{2\pi}\int_{\partial\Sigma}\left(\pi\lambda e^{\beta X^0} + V_0 X^0 K\right)\,, \tag{3.2}$$

where K is the extrinsic curvature which integrates to 2π around the boundary of a disc. (3.2) is related to the standard boundary Liouville theory with $Q = b + \frac{1}{b}$:

$$\frac{1}{2\pi}\int_\Sigma \left(\partial\phi\bar\partial\phi + \frac{Q\phi R}{4}\right) + \frac{1}{2\pi}\int_{\partial\Sigma}\left(\pi\lambda e^{b\phi} + Q\phi K\right)\,, \tag{3.3}$$

by analytic continuation $X^0 \to i\phi$, $\beta \to -ib$ and $V_0 \to -iQ$. This theory was studied in [24, 25] and the analytic continuation to (3.2) was studied in [18].

In the presence of the source, the free closed string field equations can be written as

$$(Q + \bar Q)|\Psi\rangle = |B\rangle \tag{3.4}$$

where the boundary state $|B\rangle$ obeys

$$(Q + \bar Q)|B\rangle = 0\,. \tag{3.5}$$

Q here is the BRST charge in the linear dilaton background with the Virasoro operators

$$L_m = \frac{1}{2}\sum_n : \alpha^\mu_{m-n}\alpha_{n\mu} : + \frac{i}{\sqrt{2}}(m+1)V_\nu\alpha^\nu_m\,. \tag{3.6}$$

$|\Psi\rangle$ and $|B\rangle$ contain the relevant terms:

$$|B\rangle \supset \left[\tilde A_{\mu\nu}(p)\alpha^\mu_{-1}\bar\alpha^\nu_{-1} - \tilde B(p)(\bar b_{-1}c_{-1} + b_{-1}\bar c_{-1})\right] c_0^+ c_1\bar c_1|p\rangle \tag{3.7}$$

$$|\Psi\rangle \supset \left[\tilde h_{\mu\nu}(p)\alpha^\mu_{-1}\bar\alpha^\nu_{-1} - \tilde\phi(p)(\bar b_{-1}c_{-1} + b_{-1}\bar c_{-1})\right] c_1\bar c_1|p\rangle\,, \tag{3.8}$$

where $\sqrt{2}\alpha_0^\mu|p\rangle = p^\mu|p\rangle$. The equations of motion (3.4) and (3.5) lead to

$$\frac{1}{2}p\cdot(p + 2iV)\tilde h_{\mu\nu}(p) \;=\; \tilde A_{\mu\nu}(p) \tag{3.9}$$

$$\frac{1}{2}p\cdot(p + 2iV)\tilde\phi(p) \;=\; \tilde B(p) \tag{3.10}$$

$$(p^\mu + 2iV^\mu)\tilde A_{\mu\nu}(p) - p_\nu\tilde B(p) \;=\; 0 \tag{3.11}$$

under the gauge condition

$$(p^\mu + 2iV^\mu)\tilde{h}_{\mu\nu}(p) = p_\nu \tilde{\phi}(p) \,. \tag{3.12}$$

To move to position space, we define

$$f(x) \equiv \int \frac{d^D k}{(2\pi)^D} \, e^{ip_\mu x^\mu} \, f(p) \,, \tag{3.13}$$

for $f \in \{\tilde{A}_{\mu\nu}, \tilde{B}, \tilde{h}_{\mu\nu}, \tilde{\phi}\}$, and with $p_\mu = k_\mu - iV_\mu$ where k_μ is real. $\tilde{A}_{\mu\nu}$, $\tilde{B}$, $\tilde{h}_{\mu\nu}$, $\tilde{\phi}$ $(p = k - iV)$ have support as k_μ varies continuously in the $D - p$ directions transverse to the decaying brane (including the time direction), but only for $k_\mu = 0$ in the parallel directions. Equations (3.9)-(3.11) then become

$$\frac{1}{2}(-\partial^2 + 2V^\mu \partial_\mu)\tilde{h}_{\mu\nu}(x) \;=\; \tilde{A}_{\mu\nu}(x) \tag{3.14}$$

$$\frac{1}{2}(-\partial^2 + 2V^\mu \partial_\mu)\tilde{\phi}(x) \;=\; \tilde{B}(x) \tag{3.15}$$

$$(\partial^\mu - 2V^\mu)\tilde{A}_{\mu\nu}(x) \;=\; \partial_\nu \tilde{B}(x) \tag{3.16}$$

and the gauge condition is

$$(\partial^\mu - 2V^\mu)\tilde{h}_{\mu\nu}(x) = \partial_\nu \tilde{\phi}(x) \,. \tag{3.17}$$

We need to relate $\tilde{A}_{\mu\nu}$ and $\tilde{B}$ to the spacetime stress-energy tensor. Consider the string frame effective action

$$S = \int d^D x \, \sqrt{-g} e^{-2\Phi} \left[4V^2 + R + 4(\nabla\Phi)^2\right] + S_{\text{matter}} \,. \tag{3.18}$$

Varying this action with respect to the string frame metric $g_{\mu\nu}$, we obtain

$$e^{-2\Phi}\left(G_{\mu\nu} - 2g_{\mu\nu}\nabla^2\Phi + 2\nabla_\mu\nabla_\nu\Phi + 2g_{\mu\nu}(\nabla\Phi)^2 - 2V^2 g_{\mu\nu}\right) = T_{\mu\nu} \,, \tag{3.19}$$

where $T_{\mu\nu}$ is the string frame stress tensor, defined as the variation of S_{matter} w.r.t $g^{\mu\nu}$. Varying with respect to the dilaton, we obtain

$$e^{-2\Phi}\left(2R + 8V^2 - 8(\nabla\Phi)^2 + 8\nabla^2\Phi\right) = \frac{\delta S_{\text{matter}}}{\delta\Phi} \equiv U \,. \tag{3.20}$$

Expanding (3.19)–(3.20) around the linear dilaton background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,, \Phi = V_\mu x^\mu + \phi \,, \tag{3.21}$$

in the gauge

$$(\partial^\mu - 2V^\mu)h_{\mu\nu} = \partial_\nu \left(\frac{1}{2}h - 2\phi\right) \,, \tag{3.22}$$

one finds that at the linear level

$$-\frac{1}{2}\partial^2 h_{\mu\nu} + V_\lambda \partial^\lambda h_{\mu\nu} \;=\; e^{2V\cdot x}\left(T_{\mu\nu} + \frac{1}{4}\eta_{\mu\nu}U\right) \tag{3.23}$$

$$\left(-\frac{1}{2}\partial^2 + V^\mu\partial_\mu\right)\left(\frac{1}{2}h - 2\phi\right) \;=\; \frac{1}{4}e^{2V\cdot x}U \,. \tag{3.24}$$

Comparing the above two equations to (3.14)–(3.15), we find

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} \qquad \frac{1}{2}h - 2\phi = \tilde{\phi} \tag{3.25}$$

$$T_{\mu\nu} = e^{-2V\cdot x}(\tilde{A}_{\mu\nu} - \tilde{B}\eta_{\mu\nu}) \qquad \frac{1}{4}U = e^{-2V\cdot x}\tilde{B}\,. \tag{3.26}$$

Under these identifications, the gauge conditions (3.22) and (3.17) are one and the same. The conservation equation (3.16) gives a modified conservation law for the stress tensor

$$\partial^\mu T_{\mu\nu} = \frac{1}{2}V_\nu U\,. \tag{3.27}$$

The source term is due to the linear dilaton breaking spacetime translational symmetry. This equation is the same as equation (2.8), obtained in section 2.

The boundary state coefficient $\tilde{B}(x)$ is given by a disc worldsheet one point function[6]

$$\tilde{B}(x) \;= e^{V\cdot x}\int \frac{d^Dk}{(2\pi)^D}\,e^{ik\cdot x}\left\langle e^{-ik\cdot X+V\cdot X}\right\rangle. \tag{3.28}$$

(3.28) was computed in [18] by analytic continuation from the results of [24, 25]:

$$e^{-V\cdot x}\tilde{B}(x^0) = \frac{1}{\beta}\int\frac{d^{D-p}k}{(2\pi)^{D-p}}e^{ik\cdot x}\tilde{\lambda}^{ik_0/\beta}\Gamma\left(-\frac{ik_0}{\beta}\right)\Gamma(1+i\beta k_0)\,, \tag{3.29}$$

where

$$\tilde{\lambda} \equiv \pi\lambda/\Gamma(1+\beta^2)\,. \tag{3.30}$$

Adopting the convention that $t = x^0$, we derive an explicit expression for $\tilde{B}$ as follows:

$$\begin{aligned} e^{-V\cdot x}\tilde{B}(t) &= \frac{1}{\beta}\int\frac{dk_0}{2\pi}e^{ik_0t}\tilde{\lambda}^{ik_0/\beta}\int dq\; e^{-q}\; q^{-ik_0/\beta-1}\int ds\; e^{-s}\; s^{ik_0\beta} \\ &= \int\frac{dk_0}{2\pi}e^{ik_0t}\tilde{\lambda}^{ik_0/\beta}\int dt'\; e^{-e^{\beta t'}}\; e^{-ik_0t'}\int ds\; e^{-s}\; s^{ik_0\beta} \\ &= \int ds\exp\left(-s-\tilde{\lambda}e^{\beta t}s^{\beta^2}\right), \end{aligned} \tag{3.31}$$

where we have suppressed the transverse delta functions projecting onto the brane. $\tilde{A}_{\mu\nu}$ is a one point function of a descendant of the primary in (3.28). It must be diagonal, since the total CFT describing the string is a direct sum of the CFTs describing each coordinate, and a left–right level matching condition must be satisfied for each spacelike CFT separately for a nonzero one point function. A simple way to compute $\tilde{A}_{\mu\nu}$ is to use the conservation law (3.11). Starting with the 00 component, $\tilde{A}_{00}$:

$$\tilde{A}_{00}(p) = -\frac{k_0 - iV_0}{k_0 + iV_0}\tilde{B}(p)\,, \tag{3.32}$$

[6] We note that the correlators on the right hand side contain a factor of $e^{-V\cdot X}$ arising from the worldsheet action (3.2).

JHEP11(2003)042

we obtain

$$e^{-V\cdot x}\tilde{A}_{00}(t) = \frac{1}{\beta}\int \frac{dk_0}{(2\pi)} e^{ik_0 t}\frac{-k_0+iV_0}{k_0+iV_0}\tilde{\lambda}^{ik_0/\beta}\Gamma\left(-\frac{ik_0}{\beta}\right)\Gamma(1+i\beta k_0)\,. \tag{3.33}$$

Write

$$\frac{-k_0+iV_0}{k_0+iV_0} = 2\left(\int_0^1 dy\, y^{ik_0/V_0}\right) - 1\,. \tag{3.34}$$

Then, through a derivation similar to that for $\tilde{B}$,

$$e^{-V\cdot x}\tilde{A}_{00}(t) = \int ds\left[2\int_0^1 dy\;\exp\left(-s-\tilde{\lambda}e^{\beta t}s^{\beta^2}y^{-\beta/V_0}\right) - \exp\left(-s-\tilde{\lambda}e^{\beta t}s^{\beta^2}\right)\right]. \tag{3.35}$$

In addition, one easily finds that $\tilde{A}_{ii} = -\tilde{B}$.

Since T_{ab} is zero for transverse directions a, b, we focus below on the longitudinal directions i, j. An explicit expression for $T_{\mu\nu}$ follows from above, again suppressing the transverse delta function projecting onto the brane:

$$\begin{aligned} T_{00}(x) &= e^{-2V\cdot x}(\tilde{A}_{00}+\tilde{B}) \\ &= 2e^{-V\cdot x}\int_0^1 dy\int_0^\infty ds\,\exp\left(-s-\tilde{\lambda}e^{\beta t}s^{\beta^2}y^{\gamma}\right) \end{aligned} \tag{3.36}$$

and

$$T_{ij} = -2e^{-V\cdot x}\delta_{ij}\int_0^\infty ds\,\exp\left(-s-\tilde{\lambda}e^{\beta t}\,s^{\beta^2}\right). \tag{3.37}$$

Also, we obtain the dilaton source term

$$U = 4e^{-V\cdot x}\int_0^\infty ds\,\exp\left(-s-\tilde{\lambda}e^{\beta t}\,s^{\beta^2}\right). \tag{3.38}$$

In the far past, before the brane decays, we find that

$$T_{00}(t\to-\infty) = 2e^{-V\cdot x} \tag{3.39}$$

$$T_{ij}(t\to-\infty) = -2e^{-V\cdot x}\delta_{ij}\,. \tag{3.40}$$

In the far future, after the brane has decayed,

$$T_{00}(t\to\infty) \sim e^{-V_i x^i} \tag{3.41}$$

$$T_{ij}(t\to\infty) \sim e^{-\beta t - V_i x^i}\delta_{ij}\,. \tag{3.42}$$

Note that for $t\to-\infty$ the components of the stress tensor of the D-brane scale as $1/g_s(t)$. This is expected before the tachyon condenses. For $t\to\infty$, T_{00} becomes *independent* of time, while T_{ij} decays to zero. Thus we have tachyon dust in the far future. The fact that, despite the time dependence of the string coupling, the energy density becomes constant and the pressure goes to zero corroborates the suggestion [13, 15, 16] that this tachyon dust is a gas of massive closed strings, whose energy is not affected by a change in the dilaton the way the energy of a D-brane is. The spatial dependence of the energy density for $t \to \infty$ can be explained from the assumption that the initial D-brane energy is converted to the tachyon dust during a time period around $x^0 = 0$. This initial energy profile is imprinted in the tachyon dust and subsequently unchanged.

Note that the dust energy is still proportional to $1/g_s^0$ with g_s^0 the constant part of string coupling. This implies that the (order g_s^0) energy in closed string emission computed from the annulus in the next section is not sufficient to account for the energy in the dust. So, if the tachyon dust is indeed made of massive closed strings, in this context their origin can not be understood from the annulus.

The Lorentz frame we have discussed so far is a natural one because the brane decay occurs simultaneously in all space. In the following sections we shall also see that it is the simplest frame for CFT calculations. However, for a spacelike dilaton $V_\mu V^\mu \equiv |V|^2 > 0$ there is a timelike Killing symmetry which is obscured in this frame. For simplicity let us assume that the dilaton gradient points in the x^1 direction so that

$$\Phi(x) = V_0 x^0 + V_1 x^1 \qquad T(x) = e^{\beta x^0}\,, \tag{3.43}$$

with $V_1 > 0$. Then the dilaton background has a timelike Killing vector

$$K = \frac{1}{|V|}(V^1 \partial_0 + V^0 \partial_1)\,. \tag{3.44}$$

This allows for the construction of a conserved energy-momentum four-vector

$$j_\nu = K^\mu T_{\mu\nu}\,, \qquad \partial_\mu j^\mu = 0\,. \tag{3.45}$$

Since $T_{0i} = 0$ in the frame (3.43), the tachyon dust has no momentum there. Clearly if $V_0 > 0$, the dust is moving towards strong coupling; if $V_0 < 0$, it is moving towards weak coupling.

Of particular interest is the case of a normalizable tachyon, in the sense discussed in section 2, as the physical decay of the brane would presumably proceed via such normalizable modes. It is natural to consider this issue in the Lorentz frame

$$\Phi(\tilde{x}) = |V|\tilde{x}^1 \qquad T(\tilde{x}) = \exp\left(\frac{\beta(V_1\tilde{x}^0 - V_0\tilde{x}^1)}{|V|}\right), \tag{3.46}$$

in which the dilaton is purely spatial and energy is conserved. For the tachyon to be normalizable in this frame we must have

$$2\beta V_0 = -V^2 < 0\,. \tag{3.47}$$

If we consider a spatially oscillating profile, β becomes smaller but the sign restriction on V_0 remains. Thus, if we demand normalizability, the tachyon dust slides towards weak coupling. At any given fixed spatial point (i.e. fixed value of the string coupling) in this frame, the energy decreases exponentially with time, and the dust disappears. This is illustrated in figure 1.

JHEP11(2003)042

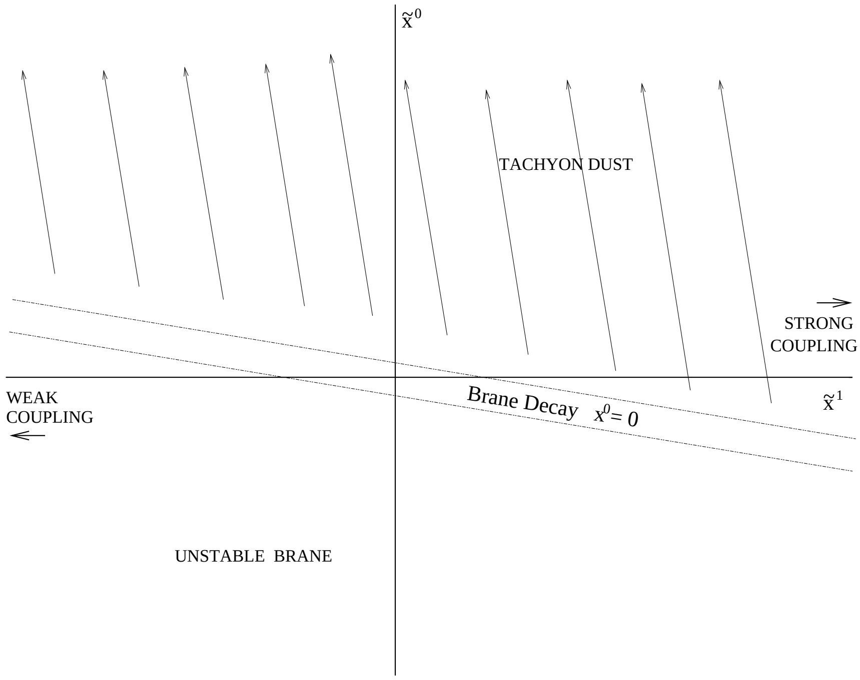

Figure 1: Brane decay in the frame in which the dilaton is time-independent but grows with $\tilde{x}^1$. In this frame, the tachyon dust produced by a normalizable decay mode drifts towards weak coupling with constant velocity. Both the energy density of the dust and the energy density of the brane which produced it decrease exponentially towards the strong coupling region.

4. Closed string emission

In this section, we will define and compute the annulus diagram, extract its imaginary part, compare with the closed string emission computed in [13] and discuss the various divergences. As in section 3, we boost into a frame where the tachyon profile is $\exp(\beta X^0)$, and take an arbitrary linear dilaton with $V_0 = \beta - 1/\beta$.

4.1 Partition function

We would like to compute the open string partition function, which in the closed string channel is written as

$$Z(\tilde{q}) = \langle B|\tilde{q}^{L_0 - \frac{D-2}{24}}|B\rangle \, . \tag{4.1}$$

This expression factorizes into the time-like and the space-like part. We will first study the time-like part in detail. It can be written as [25]

$$Z^{X^0}(\tilde{q}) = \int \frac{d\omega}{2\pi} |U(\alpha)|^2 \chi_\alpha(\tilde{q}) \, , \tag{4.2}$$

where $U(\alpha)$ is the one-point function of $e^{-2i\alpha X^0}$ on the disc. The operator $e^{-2i\alpha X^0}$ must be dressed, so $2\alpha = \omega + iV_0$, and the integration is over all real frequencies ω. This amounts to including only oscillating closed strings which behave as $e^{i\omega X^0}$ rather than $e^{\omega X^0}$. χ_α is the character,

$$\chi_\alpha(q) = \eta^{-1}(q)\; q^{-(\alpha - iV_0/2)^2} \, . \tag{4.3}$$

JHEP11(2003)042

The bulk one point function for the insertion of a dressed closed string vertex operator is [24, 18]

$$U(\alpha) \equiv \langle e^{(V_0 - i\omega)X^0} \rangle = -\frac{i}{\beta}\tilde{\lambda}^{\frac{i\omega}{\beta}}\Gamma(1+i\omega\beta)\Gamma\left(-\frac{i\omega}{\beta}\right), \tag{4.4}$$

and we obtain

$$|U(\alpha)|^2 = \frac{\pi^2}{\sinh(\pi\omega/\beta)\sinh(\pi\omega\beta)}. \tag{4.5}$$

(4.2) may be now written as

$$Z^{X^0}(\tilde{q}) = \int_{-\infty}^{\infty} \frac{d\omega}{2} \frac{\pi\ \chi_{(\omega+iV_0)/2}(\tilde{q})}{\sinh(\pi\beta\omega)\sinh(\pi\omega/\beta)}. \tag{4.6}$$

Unfortunately (4.6) is ill-defined because the character goes as $q^{-\omega^2}$ for large ω corresponding to the negative L_0 eigenvalues for timelike modes. Our strategy will be to formally manipulate (4.6) into a form which is well-defined, which will then be adopted as the definition of Z^{X^0}.

We begin with the observation that the "oscillating" closed string characters can be expressed as a modular transform of non-oscillating open string characters (which are never on-shell):

$$\chi_\alpha(\tilde{q}) = \sqrt{2}\int_{-\infty}^{\infty} d\nu \cosh\left(4\pi\left(\alpha - \frac{iV_0}{2}\right)\nu\right)\chi_{i(\nu+V_0/2)}(q), \tag{4.7}$$

where

$$q = e^{-\frac{\pi^2}{t}}, \qquad \tilde{q} = e^{-4t} \tag{4.8}$$

are modular transforms of one another. The r.h.s. of (4.6) then becomes

$$\frac{\pi}{\sqrt{2}}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\nu \left(\frac{\cosh(2\pi\omega\nu)}{\sinh(\pi\beta\omega)\sinh(\pi\omega/\beta)}\right)\chi_{i(\nu+V_0/2)}(q). \tag{4.9}$$

The expression above has a double pole at $\omega = 0$, which can be taken care of by a subtraction[7]

$$\frac{\pi}{\sqrt{2}}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\nu \left(\frac{\cosh(2\pi\omega\nu)}{\sinh(\pi\beta\omega)\sinh(\pi\omega/\beta)} - \frac{1}{\pi^2\omega^2}\right)\chi_{i(\nu+V_0/2)}(q). \tag{4.10}$$

A convergent expression for Z^{X^0} can now be found using the integral representation of the q-gamma function [24]

$$\partial_x \ln S_\beta(x) = -\int_{-\infty}^{\infty} dt \left(\frac{\cosh((2x-Q_\beta)t)}{2\sinh\beta t\sinh\frac{t}{\beta}} - \frac{1}{2t^2}\right) \tag{4.11}$$

(where $Q_\beta \equiv \beta + \frac{1}{\beta}$) and interchanging the integration order in (4.10). This gives our definition of Z^{X^0} in terms of the convergent integral

$$Z^{X^0}(q) \equiv \sqrt{2}\int_{-\infty}^{\infty} d\nu \chi_{i(\nu+V_0/2)}(q)\partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2} - \nu\right). \tag{4.12}$$

[7]The same subtraction was used in [25] to regulate the partition function; it not change the value of the integral, since a double pole has no residue.

The function $S_\beta(\frac{Q_\beta}{2}-\nu)$ has simple poles at $\nu=(m+1/2)\beta+\frac{n+\frac{1}{2}}{\beta}$ for non-negative integer m,n and simple zeroes for negative integer m,n. We take the contour in (4.12) to go just above the real axis for positive ν and just below for negative ν.

Z^{X^0} acquires an imaginary part from integrating in the ν plane past the poles

$$\mathrm{Im}\, Z^{X^0}(q) = 2\sqrt{2}\pi\ \eta^{-1}(q) \sum_{m,n=0}^{\infty} e^{-((n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta)^2\frac{\pi^2}{t}}\,. \tag{4.13}$$

The space-like part (including ghosts) of the partition function is standard

$$\begin{aligned} Z^{X^i+bc}(q) &= V_p \int \frac{d^{D-1-p}k_\perp}{(2\pi)^{D-1-p}} \tilde{q}^{k_\perp^2/4}\eta^{3-D}(\tilde{q}) \\ &= \frac{\mathcal{N}_p^{-2}V_p}{2\sqrt{2}\pi t}\int \frac{d^pk_\parallel}{(2\pi)^p}\ q^{k_\parallel^2}\eta^{3-D}(q)\,, \end{aligned} \tag{4.14}$$

where $\mathcal{N}_p=\pi^{(D-4)/4}(2\pi)^{(D-2)/4-p}$ is the normalization of the boundary state, same as for the non-decaying brane. Putting the two things together, we obtain

$$\begin{aligned} Z(q) &= \mathcal{N}_p^2 Z^{X^i+bc}Z^{X^0} \\ &= \frac{V_p}{2\pi t}\int_{-\infty}^{\infty} d\nu \int \frac{d^pk_\parallel}{(2\pi)^p} q^{\nu^2+k_\parallel^2}\eta^{2-D}(q)\partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2}-\nu\right). \end{aligned} \tag{4.15}$$

The imaginary part from the poles is

$$\mathrm{Im}\, Z(q) = \frac{V_p}{t}\int \frac{d^pk_\parallel}{(2\pi)^p}\ q^{k_\parallel^2}\ \eta^{2-D}(q) \sum_{m,n=0}^{\infty} e^{-((n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta)^2\frac{\pi^2}{t}}\,. \tag{4.16}$$

Without the linear dilaton, $\beta=1, V_\mu=0$, and the above expression simplifies to

$$\mathrm{Im}\, Z(q) = \frac{V_p}{t}\int \frac{d^pk_\parallel}{(2\pi)^p}\ q^{k_\parallel^2}\ \eta^{-24}(\tilde{q}) \sum_{n=1}^{\infty} n e^{-n^2\frac{\pi^2}{t}}\,. \tag{4.17}$$

In subsection 4.3, we shall see that this agrees with the expected answer for closed string emission. But first, we want to consider whether the annulus diagram might have any other contributions to the imaginary part.

4.2 Annulus diagram

In principle the full imaginary part of the annulus could contain additional terms associated with divergences in the integration of the partition function $Z(q)$ over the modulus. In the computation of, for example, open string pair production in an electric field, such imaginary parts arise near poles from on-shell open string states. We will not find such imaginary parts here but the form of the integrated partition function is nevertheless illuminating.

Writing $q=e^{-s}$ $(s=\pi^2/t)$, adding an integration over s and writing the η functions as a sum over open string oscillators N yields

$$\begin{aligned} \mathbf{Z} &= \int ds\ Z\left(\frac{is}{2\pi}\right) \\ &= V_p\int \frac{ds}{2\pi s}\int_{-\infty}^{\infty} d\nu \int \frac{d^pk_\parallel}{(2\pi)^p}\sum_N e^{-s(\nu^2+k_\parallel^2+N-1)}\partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2}-\nu\right). \end{aligned} \tag{4.18}$$

We will evaluate this by defining $\mathbf{Z} \equiv V_p \int \frac{d^p k_\parallel}{(2\pi)^p} \mathbf{Z}(k_\parallel^2)$ so that

$$-\frac{d}{dk_\parallel^2}\mathbf{Z}(k_\parallel^2) = \int \frac{ds}{2\pi} \int_{-\infty}^{\infty} d\nu \sum_N e^{-s(\nu^2+k_\parallel^2+N-1)} \partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2} - \nu\right)$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \sum_N \frac{1}{\nu^2+k_\parallel^2+N-1} \partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2} - \nu\right). \qquad (4.19)$$

The ν integration can now be done by closing the contour in the upper half-plane. The result picks up poles from ∂S_β, as well as a contribution from the pole at $\nu = i\sqrt{k_\parallel^2 + N - 1}$ corresponding to on-shell open strings. One finds

$$-\frac{d}{dk_\parallel^2}\mathbf{Z}(k_\parallel^2) = \sum_N \sum_{m,n=0}^{\infty} \frac{i}{((n+\frac{1}{2})\beta + (m+\frac{1}{2})/\beta)^2 + k_\parallel^2 + N - 1} +$$
$$+ \sum_N \frac{i}{2\nu} \partial_\nu \ln S_\beta\left(\frac{Q_\beta}{2} - \nu\right)|_{\nu = i\sqrt{k_\parallel^2+N-1}} - \frac{\pi}{2}, \qquad (4.20)$$

where the last term comes from subtracting the contour at infinity. Integrating with respect to $k_\parallel^2$ gives

$$\mathbf{Z}(k_\parallel^2) = V_p \sum_N i\left[\sum_{m,n=0}^{\infty} -\ln\left(\left(\left(n+\frac{1}{2}\right)\beta + \frac{(m+1/2)}{\beta}\right)^2 + k_\parallel^2 + N - 1\right) + \right.$$
$$\left. + \ln S_\beta\left(\frac{Q_\beta}{2} - i\sqrt{k_\parallel^2+N-1}\right) - \frac{i\pi k_\parallel^2}{2} + \text{constant}\right], \qquad (4.21)$$

where the leading large $k_\parallel^2$ behavior of S_β just cancels the $\frac{i\pi k_\parallel^2}{2}$ term. $\mathbf{Z}$ is then

$$\mathbf{Z} = \sum_N V_p \int \frac{d^p k_\parallel}{(2\pi)^p}\left[-i\sum_{m,n=0}^{\infty} \ln\left[\left(\left(n+\frac{1}{2}\right)\beta + \frac{(m+1/2)}{\beta}\right)^2 + k_\parallel^2 + N - 1\right] + \right.$$
$$\left. + i\ln S_\beta\left(\frac{Q_\beta}{2} - i\sqrt{k_\parallel^2+N-1}\right) + \frac{\pi k_\parallel^2}{2} + \text{constant}\right]. \qquad (4.22)$$

The first part reproduces (4.16). Using the identity [24] $S_\beta(x)S_\beta(Q_\beta - x) = 1$, we see that the second term, which arises at the pole for on-shell open string states, has no imaginary part. This suggests that the imaginary part does not contain an extra contribution from open string pair production.

Divergences remain in (4.22) from both closed and open string tachyons. Presumably the divergence associated with the closed string tachyon will be absent in the superstring. But even there, an unstable brane always has an open string tachyon. The divergence arises because there is no vacuum state for the incoming tachyon. Quantum fluctuations prevent one from perching the tachyon at the maximum of its potential all the way to the infinite past. Therefore initial conditions must be imposed at some finite time, which complicates the analysis. A more physical context to study this is in the full s-brane, in which the tachyon remains only momentarily at its maximum. For the full s-brane in the superstring we do not expect any divergences from tachyons, if the initial conditions are chosen correctly.

JHEP11(2003)042

4.3 Closed string emission and its divergences

Performing a modular transform on (4.16), and integrating over $s = \pi^2/t$, we can write it in the form

$$\operatorname{Im}\mathbf{Z} = V_p \sum_{n,m=0}^{\infty} \int_0^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{1}{2}p}}\ e^{-\left((n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta\right)^2 s} \frac{1}{\eta^{D-2}\left(\frac{is}{2\pi}\right)} . \tag{4.23}$$

For the special case, $V_\mu = 0, \beta = 1$, this simplifies to

$$\operatorname{Im}\mathbf{Z} = V_p \sum_{n=1}^{\infty} n \int_0^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{1}{2}p}}\ e^{-n^2 s}\ \frac{1}{\eta^{24}\left(\frac{is}{2\pi}\right)} . \tag{4.24}$$

Closed string emission (without the linear dilaton) was computed in [13]. In the appendix, we review that computation and extend it to the linear dilaton background. The answers we obtain agree precisely with the above expressions for $\operatorname{Im}\mathbf{Z}$. Thus, we can interpret the imaginary part of the annulus diagram as the amplitude for closed string emission.

We are interested in its divergences, begining our analysis with the $V_\mu = 0$ case. The expression (4.24) above has potential for both open string IR divergences (large s) and UV divergences (small s).

For large s, we have that $\eta(is/2\pi) \sim \exp(-s/24)$ plus exponentially suppressed corrections, thus, in the IR, the expression is divergent if $p = 0$ and convergent for larger p. This corresponds to a UV divergence in the emission of highly massive closed strings. It can be understood as originating from massless open strings stretching between two sD-branes [13].

Similar expressions exist for the expectation value of powers of energy in the closed string emission. These expressions contain an extra power of s for every power of energy. Thus, the expectation value of E^m will be divergent for $p \leq 2m$. This indicates that the system becomes strongly coupled for any p.

In the UV (at small s) the expression is approximately

$$V_p \int_0 \frac{ds}{2s^2} \frac{1}{(4\pi s)^{\frac{1}{2}p}} \left(\frac{s}{2\pi}\right)^{12} \left(e^{\frac{4\pi^2}{s}} + 24 + \dots\right) . \tag{4.25}$$

After we throw out the highly divergent first term (which is due to the closed string tachyon), there is still a power-law UV divergence for $p \geq 22$.

These divergences signal the breakdown of string perturbation theory. Let us now examine the expression including the linear dilaton, (4.23).

The UV region of the closed string emission is $E \to \infty$, or equivalently $s \to \infty$ (open string IR). In that limit, we obtain

$$\operatorname{Im}\mathbf{Z} \sim V_p \int^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{1}{2}p}}\ e^{-\frac{1}{4}(\beta+1/\beta)^2 s}\ e^{\frac{D-2}{24}s} . \tag{4.26}$$

The exponent is negative as long as $V_i \neq 0$, since

$$-\frac{1}{4}\left((\beta+1/\beta)\right)^2 + \frac{D-2}{24} = -\frac{V_i^2}{4} , \tag{4.27}$$

where V_i is the spacelike part of the dilaton gradient. There is no UV divergence as long as the dilaton is not precisely aligned with the timelike direction of the decay. This is of course always the case if the dilaton gradient is spacelike. Since we would like to make a Lorentz invariant statement about finite production for all exponential decay modes of the tachyon $e^{t_\mu X^\mu}$, we shall restrict our claims to spacelike dilaton gradients.

Expression (4.23) can be interpreted as the partition function for open strings stretched between an array of sD-branes at imaginary time. These are located at $X^0 = \sigma(n+1/2)\beta$ for $n \geq 0$ and at $X^0 = -\sigma(m+1/2)/\beta$ for $m \geq 0$, with $\sigma = \pm i$. A UV divergence in closed string emission transforms, under a modular transformation, into an IR divergence arising from a massless open string. Here, the mass of the lightest open string state is

$$m^2_{\text{lightest}} = \left(\frac{1}{2}(\beta + 1/\beta)\right)^2 - \frac{D-2}{24} = \frac{V_i^2}{4}\,, \tag{4.28}$$

in agreement with above discussion.

There are also potential UV divergences in the open string channel. These appear in the expansion at small s:

$$V_p \int_0 \frac{ds}{2s^2} \frac{1}{(4\pi s)^{\frac{1}{2}p}} \left(\frac{s}{2\pi}\right)^{\frac{D-2}{2}} \left(e^{\frac{(D-2)\pi^2}{6s}} + (D-2)e^{\frac{(D-26)\pi^2}{6s}} + \dots\right). \tag{4.29}$$

Hence, for $D < 26$, a spacelike dilaton, the UV divergence (ignoring the first term, due to the closed string tachyon) is gone as well.

4.4 Open-closed string duality

For simplicity, in this subsection the linear dilaton is set to zero ($\beta = 1$).

In subsection 4.1, we wrote the partition function as an integral over non-oscillating open string states

$$Z^{X^0}(q) \equiv \int_{-\infty}^{\infty} d\nu \sqrt{2}\,\pi\nu \cot(\pi\nu)\, \chi_{i\nu}(q)\,. \tag{4.30}$$

In general, the partition function can be written as

$$Z^{X^0}(q) \equiv \int_{-\infty}^{\infty} d\nu\, N(\alpha)\, \chi_\alpha(q)\,, \tag{4.31}$$

where $N(\alpha)$ the spectral density of open strings. $N(\alpha)$ is not known, but (just as for ordinary Liouville theory) we might be able to gain some insight into it by considering the mini-superspace approximation.

In the mini-superspace approximation, the annulus diagram can be written as

$$\mathbf{Z} = V_p \int_0^\infty \frac{ds}{s} \frac{1}{\eta^{24}(is/2\pi)} \frac{1}{(4\pi s)^{\frac{p}{2}}} \operatorname{Tr} e^{sH}\,, \tag{4.32}$$

where H is the hamiltonian for a particle moving in one-dimension

$$H = -\partial_t^2 - \lambda e^t\,. \tag{4.33}$$

Notice that this looks real, but can develop imaginary parts due to divergences.

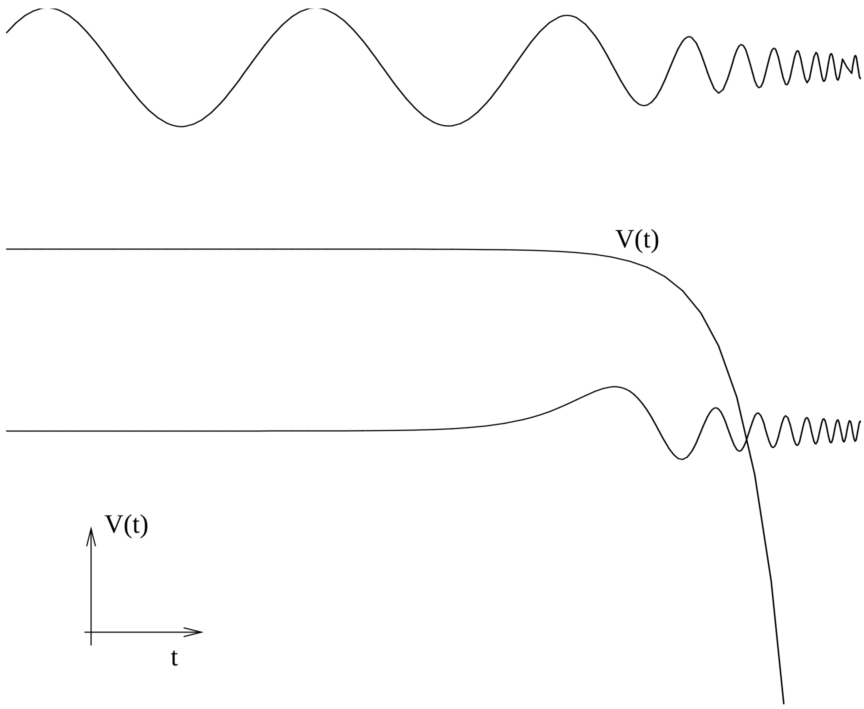

Figure 2: In the unbounded from below potential V(t), there are two kinds of eigenstates. Those with positive total energy oscillate in the far past, and correspond to on-shell strings propagating on the brane. Those with negative energy decay exponentially in the far past and are always off-shell. Poles in the spectral density of these unphysical states correspond to closed string emission.

The problem thus reduces to a problem in quantum mechanics [4, 12]

$$-\partial_t^2\phi(t) + V(t)\phi(t) = \omega^2\phi(t)\,, \quad V(t) = -\lambda e^t\,, \tag{4.34}$$

where t is the zero mode of X^0. The solutions to the above equation are Bessel functions, $\phi(t) \sim J_{2i\omega}(2\sqrt{\lambda}e^{t/2})$. ω can either be real, or equal to $-i\nu$ with ν real and positive.

In the first branch (real ω), the integral over s in equation (4.32) will diverge when the mode under consideration is on-shell. Analytic continuation controlling this divergence will give $\mathbf{Z}$ an imaginary part, corresponding to open string pair production.

In the second branch, open strings cannot go on shell. However, looking at equation (4.30) and comparing to (4.31), we realize that the spectral density on this branch has poles as a function of ν. These poles correspond to production of on-shell closed strings, as computed in subsection 4.1.

It is not clear that in a given boundary CFT (determined by initial conditions for both closed and open strings) both these branches contribute to the annulus.

5. Open string pair production

In this section we compute the pair production of open strings in a linear dilaton background along the lines discussed in [4, 18, 12].

Open strings are in general pair produced on a decaying brane because they are governed by a time-dependent hamiltonian. This implies that the *in* and *out* vacua are not

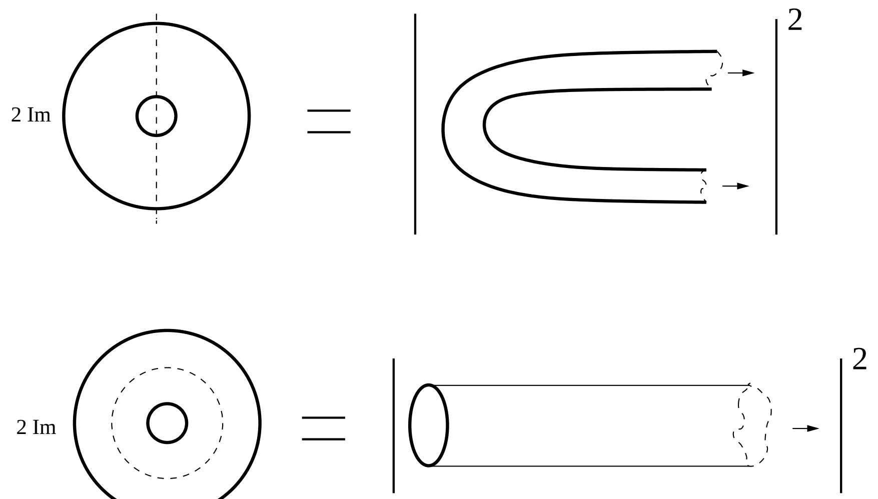

Figure 3: The annulus can be interpreted as the square of a closed string one-point function or open string two point function, depending on how it is sliced. Corresponding imaginary parts from on-shell closed string emission or open string pair production can appear in different kinematical regions of the diagram.

the same. The vacuum decay amplitude is

$$W = -\mathrm{Re}\ln\langle \mathrm{out}|\mathrm{in}\rangle\,. \tag{5.1}$$

We work in an approximation in which the open strings propagate freely on the brane so that (5.1) is a one loop computation. The interpretations of the annulus as either open or closed string production correspond to different slicings, as illustrated in figure 3. (5.1) is the formula used by Schwinger to compute pair production in an electric field. Note however that in the latter case there is an additional volume divergence coming from the time direction, whereas here the open string creation is localized in time and there is no such divergence.

The *in* and *out* vacua are related by a Bogolubov transformation

$$|\mathrm{out}\rangle = \prod_{k_{\|}} (1-|\gamma_{k_{\|}}|^2)^{1/4} e^{-\frac{1}{2}\gamma^*_{k_{\|}}(a^{in\dagger}_{k_{\|}})^2} |in\rangle\,, \tag{5.2}$$

where $\gamma_{k_{\|}}$ and $a^{in\dagger}_{k_{\|}}$ are the Bogolubov coefficients and the creation operators associated to an open string mode with longitudinal spatial momentum $k_{\|}$. (We follow the notation of [4, 18, 12].) It follows that

$$\frac{W}{V_p} = -\frac{1}{4}\sum_N \int \frac{d^p k_{\|}}{(2\pi)^p} \ln(1-|\gamma_{k_{\|}}|^2)\,, \tag{5.3}$$

where

$$\omega^2 = k_{\|}^2 + N - \frac{D-2}{24}\,. \tag{5.4}$$

The Bogolubov coefficients γ are conjecturally related [18] to the boundary two point function of dressed open string operators. At large ω (and hence small γ_ω):

$$|\gamma_\omega|^2 \sim \left| d\left(\omega + \frac{iV_0}{2}\right)\right|^2 , \tag{5.5}$$

which then determines the convergence of (5.3) at large ω. From formulae in [18, 24], the norm of the two point function is[8]

$$\left| d\left(\omega + \frac{iV_0}{2}\right)\right|^2 = \left| \frac{\beta S_{i\beta}(2\omega + iV_0)\Gamma(1-2i\omega/\beta)\Gamma(2i\beta\omega)}{2\pi S^2_{i\beta}(\omega + iV_0/2)}\right|^2 . \tag{5.6}$$

$S_{i\beta}(x)$ is the ratio of Barnes double gamma functions employed in [24].[9] It has an expansion for large x with $\pm\mathrm{Im}\, x > 0$ as

$$\ln S_{i\beta}(x) = \pm \frac{i\pi}{2}\left[x^2 - iV_0 x + O(x^0)\right] . \tag{5.7}$$

Similarly for large positive x Stirling's formula gives

$$\mathrm{Re} \ln \Gamma(ix) = -\frac{\pi x}{2} + O(x^0) . \tag{5.8}$$

We wish to find the asymptotic behavior of (5.6) for large ω. If $\beta < 1$, $V_0 < 0$ and we should use the lower sign in (5.7) which yields

$$\left| d\left(\omega + \frac{iV_0}{2}\right)\right|^2 = e^{-\frac{4\pi\omega}{\beta} + O(\omega^0)} . \tag{5.9}$$

On the other hand for $\beta > 1$ we should use the upper sign which yields instead

$$\left| d\left(\omega + \frac{iV_0}{2}\right)\right|^2 = e^{-4\pi\omega\beta + O(\omega^0)} . \tag{5.10}$$

It follows that for large ω and $\beta < 1$,

$$\frac{W}{V_p} \sim \sum_N \int d^p k_{\|} e^{-\frac{4\pi\omega}{\beta}} \sim \int d\omega e^{\frac{\omega}{T_H} - \frac{4\pi\omega}{\beta}} \sim \int d\omega e^{2\pi\omega\sqrt{\frac{D-2}{6}} - \frac{4\pi\omega}{\beta}} , \tag{5.11}$$

where we have used expression (2.10) for T_H. The corresponding expression for $\beta > 1$ involves $\beta \to 1/\beta$. This gives the production of highly massive open string pairs. The exponent here is more negative than the one we saw earlier in equation (4.26) for the highly massive *closed* strings. Hence the linear dilaton background renders finite both the closed string emission and the open string pair production rates.

Without the dilaton, $\beta = 1$ and $D = 26$, and the two exponentials cancel. There is then a power divergence for $p \geq 23$.

[8]This follows from (4.7), (4.10) and (4.31) of [18] upon substitution of $b = i\beta$, $Q = iV_0$.

[9]In general $S_{i\beta}$ is ill defined for real β as discussed in [18] but the quantities we compute here have a smooth limit as β approaches the real axis.

JHEP11(2003)042

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A. Closed string emission

In this appendix, we will compute closed string emission during brane decay in various linear dilaton theories. We begin with a review of [13] for closed string emission during brane decay (or half s-brane) without a dilaton.[10] These formulae will be generalized to a linear dilaton background.

Flat space brane decay is described by a TBL string worldsheet CFT with an action for the timelike coordinate X^0

$$-\frac{1}{2\pi}\int_{\Sigma}\partial X^0\partial X^0+\frac{\lambda}{2}\int_{\partial\Sigma}e^{X^0}\,, \tag{A.1}$$

where we have set $\alpha'=1$. The one point function for this theory is [24, 18]

$$U\left(\alpha=\frac{\omega}{2}\right)=\langle e^{-i\omega X^0}\rangle=\tilde{\lambda}^{i\omega}\frac{\pi}{\sinh\pi\omega}\,. \tag{A.2}$$

$U(\alpha)$ is related to the number of closed strings emitted by the half s-brane via [13]

$$\frac{\bar{N}}{V_p}=\mathcal{N}_p^2\sum_s\frac{1}{2\omega_s}\left|U\left(\frac{\omega}{2}\right)\right|^2, \tag{A.3}$$

where $\mathcal{N}_p=\pi^{\frac{11}{2}}(2\pi)^{6-p}$ is a normalization constant. The sum is over all the closed strings with identical left and right moving sectors, which is equivalent to the sum over open strings. ω_s is the on-shell energy for a given closed string state,

$$\omega_s^2=k_\perp^2+4N-4\,. \tag{A.4}$$

It is useful to rewrite equation (A.3) in the open string channel, where its divergences can be most easily analyzed. Substitute equation (A.2) into (A.3) and expand to obtain

$$\frac{\bar{N}}{V_p}=\mathcal{N}_p^2\sum_N\int\frac{d^{25-p}k_\perp}{(2\pi)^{25-p}}\frac{1}{2\omega}\sum_{n=1}^{\infty}(4\pi ne^{-2\pi\omega n})\,. \tag{A.5}$$

The sum is over the Hilbert space of a single open string and the integral is over all momenta transverse to the brane. ω is a function of N and $k_\perp$ as given by equation (A.4). Now, rewrite

$$\begin{aligned}\frac{1}{2\omega}e^{-2\pi\omega n}&=\frac{1}{2\pi}\int dk_0\frac{1}{k_0^2+\omega^2}e^{2\pi ik_0n}\\&=\frac{1}{2\pi}\int dk_0\int_0^\infty dte^{-t(k_0^2+k_\perp^2+4(N-1))}e^{2\pi ik_0n}\,,\end{aligned} \tag{A.6}$$

[10]Nearly identical results apply to the full s-brane with a Hartle-Hawking-type contour prescription [13].

and perform the gaussian integrals over $k_\perp$ and k_0 to obtain

$$\frac{\bar{N}}{V_p} = C \sum_{n=1}^{\infty} n \int_o^{\infty} dt\, t^{-\frac{26-p}{2}} e^{-\frac{(\pi n)^2}{t}} \sum_N e^{-4t(N-1)}\,. \tag{A.7}$$

The sum over all open string modes N gives an eta function and the answer, after a modular transform $s = (2\pi)^2/(4t)$, is

$$\begin{aligned}\frac{\bar{N}}{V_p} &= C \sum_{n=1}^{\infty} n \int_0^{\infty} dt\ t^{-\frac{26-p}{2}} e^{-\frac{(\pi n)^2}{t}} \eta^{-24}\left(\frac{4ti}{2\pi}\right) \\ &= \sum_{n=1}^{\infty} n \int_0^{\infty} \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{1}{2}p}}\ e^{-n^2 s}\ \frac{1}{\eta^{24}\left(\frac{is}{2\pi}\right)}\,. \end{aligned} \tag{A.8}$$

C in the above is a constant, $(2\pi)^{-12-p}\pi^{24-p/2}$.

We wish to evaluate formula (A.3) for $\frac{\bar{N}}{V_p}$ in the presence of a linear dilaton. The squared one point function (4.4) is

$$\begin{aligned}\left|U\left(\frac{\omega + iV_0}{2}\right)\right|^2 &= \frac{\pi^2}{\sinh(\pi\omega/\beta)\sinh(\pi\omega\beta)} \\ &= 4\pi^2 \sum_{m,n=0}^{\infty} e^{-2\pi(n+\frac{1}{2})\omega\beta} e^{-2\pi(m+\frac{1}{2})\omega/\beta}\,. \end{aligned} \tag{A.9}$$

Adapting the result in equation (A.8) we obtain

$$\frac{\bar{N}}{V_p} = \sum_{n,m=0}^{\infty} \int_0^{\infty} \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{1}{2}p}}\ e^{-\left((n+\frac{1}{2})\beta + (m+\frac{1}{2})/\beta\right)^2 s}\ \frac{1}{\eta^{D-2}\left(\frac{is}{2\pi}\right)}\,, \tag{A.10}$$

where $D - 2 = 6(4 - V^2)$.

References

[1] M. Gutperle and A. Strominger, *Spacelike branes*, *J. High Energy Phys.* **04** (2002) 018 [hep-th/0202210].

[2] A. Sen, *Rolling tachyon*, *J. High Energy Phys.* **04** (2002) 048 [hep-th/0203211].

[3] A. Sen, *Time and tachyon*, *Int. J. Mod. Phys.* **A 18** (2003) 4869 [hep-th/0209122].

[4] A. Strominger, *Open string creation by s-branes*, hep-th/0209090.

[5] T. Okuda and S. Sugimoto, *Coupling of rolling tachyon to closed strings*, *Nucl. Phys.* **B 647** (2002) 101 [hep-th/0208196].

[6] B. Chen, M. Li and F.-L. Lin, *Gravitational radiation of rolling tachyon*, *J. High Energy Phys.* **11** (2002) 050 [hep-th/0209222].

[7] A. Sen, *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[8] A. Sen, *Tachyon matter*, *J. High Energy Phys.* **07** (2002) 065 [hep-th/0203265].

[9] N.R. Constable and F. Larsen, *The rolling tachyon as a matrix model*, *J. High Energy Phys.* **06** (2003) 017 [hep-th/0305177].

[10] F. Larsen, A. Naqvi and S. Terashima, *Rolling tachyons and decaying branes*, *J. High Energy Phys.* **02** (2003) 039 [hep-th/0212248].

[11] P. Mukhopadhyay and A. Sen, *Decay of unstable D-branes with electric field*, *J. High Energy Phys.* **11** (2002) 047 [hep-th/0208142].

[12] A. Maloney, A. Strominger and X. Yin, *S-brane thermodynamics*, *J. High Energy Phys.* **10** (2003) 048 [hep-th/0302146].

[13] N. Lambert, H. Liu and J. Maldacena, *Closed strings from decaying D-branes*, hep-th/0303139.

[14] D. Gaiotto, N. Itzhaki and L. Rastelli, *Closed strings as imaginary D-branes*, hep-th/0304192.

[15] A. Sen, *Open and closed strings from unstable D-branes*, *Phys. Rev.* **D 68** (2003) 106003 [hep-th/0305011].

[16] S. Minwalla, private communication.

[17] V. Schomerus, *Rolling tachyons from Liouville theory*, hep-th/0306026.

[18] M. Gutperle and A. Strominger, *Timelike boundary Liouville theory*, *Phys. Rev.* **D 67** (2003) 126002 [hep-th/0301038].

[19] J. McGreevy and H. Verlinde, *Strings from tachyons: the $c = 1$ matrix reloated*, hep-th/0304224.

[20] I.R. Klebanov, J. Maldacena and N. Seiberg, *D-brane decay in two-dimensional string theory*, *J. High Energy Phys.* **07** (2003) 045 [hep-th/0305159].

[21] J. McGreevy, J. Teschner and H. Verlinde, *Classical and quantum D-branes in 2D string theory*, hep-th/0305194.

[22] R.C. Myers, *New dimensions for old strings*, *Phys. Lett.* **B 199** (1987) 371.

[23] J. Polchinski, *String theory. Vol. 1. An introduction to the bosonic string*, Cambridge University Press, Cambridge 1998.

[24] V. Fateev, A.B. Zamolodchikov and A.B. Zamolodchikov, *Boundary Liouville field theory, I. Boundary state and boundary two-point function*, hep-th/0001012.

[25] J. Teschner, *Remarks on Liouville theory with boundary*, hep-th/0009138.

JHEP11(2003)042